Electronic Appendix and Supplementary Material for: Growth Regressions, Principal Components Augmented Regressions and Frequentist Model Averaging

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Appendix

Fernandez *et al.* (2001) Data

The data can be downloaded from `http://qed.econ.queensu.ca/jae/2001-v16.5/`.

Dummy variables, $0-1$ unless stated otherwise, contained in X_2: BritCol $(0, 1, \ldots, 5)$, EcoOrg, FrenchCol, OutwarOr, SpanishCol, WarDummy

Variable	Mean	StdDev	Skewness	Kurtosis	5	10	25	50	75	90	95
Confucius	0.0602	0.0046	-0.1025	2.6294	0.0524	0.0538	0.0573	0.0603	0.0634	0.0663	0.0675
EquipInv	0.1479	0.0334	0.3222	2.3018	0.0983	0.1052	0.1235	0.1440	0.1722	0.1971	0.2078
GDPsh560	-0.0167	0.0009	0.5616	3.1172	-0.0180	-0.0178	-0.0174	-0.0169	-0.0162	-0.0155	-0.0151
LifeExp	0.0008	0.0001	0.2431	2.6095	0.0006	0.0007	0.0007	0.0008	0.0009	0.0010	0.0011
Muslim	0.0123	0.0029	-0.0802	2.2564	0.0075	0.0084	0.0103	0.0122	0.0145	0.0163	0.0170
SubSahara	-0.0067	0.0030	-0.1216	2.6487	-0.0119	-0.0107	-0.0087	-0.0066	-0.0043	-0.0028	-0.0017
EthnoLFrac	0.0082	0.0018	0.7641	3.4320	0.0056	0.0060	0.0068	0.0080	0.0092	0.0108	0.0117
HighEnroll	-0.0266	0.0242	-0.7621	3.0160	-0.0757	-0.0657	-0.0404	-0.0221	-0.0096	-0.0001	0.0041
Hindu	-0.0254	0.0214	-0.3526	1.6485	-0.0617	-0.0570	-0.0435	-0.0172	-0.0066	-0.0020	-0.0007
LabForce	0.0000	0.0000	0.3144	1.5019	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LatAmerica	-0.0045	0.0022	0.0466	2.3878	-0.0081	-0.0074	-0.0061	-0.0045	-0.0028	-0.0016	-0.0009
Mining	0.0415	0.0040	0.2625	2.2272	0.0359	0.0366	0.0378	0.0419	0.0443	0.0467	0.0484
NEquipInv	0.0609	0.0073	-0.1092	2.3674	0.0481	0.0506	0.0563	0.0607	0.0667	0.0707	0.0721
PrScEnroll	0.0133	0.0046	0.0287	1.8888	0.0061	0.0070	0.0093	0.0132	0.0172	0.0191	0.0206
RuleofLaw	0.0073	0.0019	0.2168	2.5620	0.0040	0.0047	0.0059	0.0073	0.0084	0.0100	0.0107

Table 4: Mean, standard deviation, skewness, kurtosis and quantiles of the empirical coefficient distributions over all models where the respective variables are included for the Fernandez *et al.* (2001) data set.

	0	1	2	3	4	5	6	7	8	9
Equal	0.002	0.018	0.070	0.164	0.246	0.246	0.164	0.070	0.018	0.002
S-AIC	0.000	0.000	0.007	0.024	0.053	0.159	0.286	0.287	0.161	0.021
S-BIC	0.000	0.037	0.231	0.252	0.176	0.169	0.097	0.031	0.006	0.000

Table 5: Distribution of model weights over model sizes for the Fernandez *et al.* (2001) data for the three discussed weighting schemes.

Supplementary Material: Inference for Model Average Coefficients

In order to describe how to perform inference as derived in Claeskens and Hjort (2008) some further quantities need to be defined first. Denote with $e_j \in \mathbb{R}^{k_{11}+r}$ a vector with 0 entries except for 1 at position j and with $\tilde{e}_j \in \mathbb{R}^{k_{12}}$ a vector with 0 entries except for 1 at position j. Next define $\tau_{0i}^2 \in \mathbb{R}_0^+$ and $\omega_i \in \mathbb{R}^{k_{12}}$ as

$$\tau_{0i}^2 = \begin{cases} e_i' \left(\begin{bmatrix} X_{11}' \\ \tilde{X}_2' \end{bmatrix} \begin{bmatrix} X_{11} & \tilde{X}_2 \end{bmatrix} \right)^{-1} e_i, & i = 1, \ldots, k_{11} \\ 0, & i = k_{11}+1, \ldots, k_{11}+k_{12} \\ e_{i-k_{12}}' \left(\begin{bmatrix} X_{11}' \\ \tilde{X}_2' \end{bmatrix} \begin{bmatrix} X_{11} & \tilde{X}_2 \end{bmatrix} \right)^{-1} e_{i-k_{12}}, & i = k_{11}+k_{12}+1, \ldots, k_{11}+k_{12}+r \end{cases} \tag{1}$$

and

$$\omega_i = \begin{cases} X_{12}' \begin{bmatrix} X_{11} & \tilde{X}_2 \end{bmatrix} \left(\begin{bmatrix} X_{11}' \\ \tilde{X}_2' \end{bmatrix} \begin{bmatrix} X_{11} & \tilde{X}_2 \end{bmatrix} \right)^{-1} e_i, & i = 1, \ldots, k_{11} \\ \tilde{e}_{i-k_{11}}, & i = k_{11}+1, \ldots, k_{11}+k_{12} \\ X_{12}' \begin{bmatrix} X_{11} & \tilde{X}_2 \end{bmatrix} \left(\begin{bmatrix} X_{11}' \\ \tilde{X}_2' \end{bmatrix} \begin{bmatrix} X_{11} & \tilde{X}_2 \end{bmatrix} \right)^{-1} e_{i-k_{12}}, & i = k_{11}+k_{12}+1, \ldots, k_{11}+k_{12}+r \end{cases} \tag{2}$$

Further, we need the block of the information matrix for the coefficient vector β in the full regression including $[X_{11}\ X_{12}\ \tilde{X}_2]$, given in partitioned format by

$$\mathcal{I} = \frac{1}{N\sigma^2} \begin{bmatrix} X_{11}'X_{11} & X_{11}'X_{12} & X_{11}'\tilde{X}_2 \\ X_{12}'X_{11} & X_{12}'X_{12} & X_{12}'\tilde{X}_2 \\ \tilde{X}_2'X_{11} & \tilde{X}_2'X_{12} & \tilde{X}_2'\tilde{X}_2 \end{bmatrix}. \tag{3}$$

In the computations the unknown quantity σ^2 is replaced by the estimate from the full model. The 2-2 block of $\mathcal{I}^{-1}$, i.e. the block at the position of $X_{12}'X_{12}$ in $\mathcal{I}$, is denoted by $(\mathcal{I}^{-1})_{(2,2)}$. Next denote the set of indices of variables of X_{12} included in $\mathcal{M}_j$ as S_j and its cardinality by $|S_j|$, i.e. $S_j = \left\{ i_1, \ldots, i_{|S_j|} \right\} \subseteq \{1, \ldots, k_{12}\}$. Without loss of generality we index the model excluding all variables of X_{12} as $\mathcal{M}_1$. For all models except $\mathcal{M}_1$ define

$$\pi_j = \begin{bmatrix} \tilde{e}_{i_1}' \\ \vdots \\ \tilde{e}_{i_{|S_j|}}' \end{bmatrix} \in \mathbb{R}^{|S_j| \times k_{12}} \tag{4}$$

and $G(j) = \pi_j' \left(\pi_j \left((\mathcal{I}^{-1})_{(2,2)} \right)^{-1} \pi_j' \right)^{-1} \pi_j \left((\mathcal{I}^{-1})_{(2,2)} \right)^{-1} \in \mathbb{R}^{k_{12} \times k_{12}}$ for $j = 2, \ldots, 2^{k_{12}}$. For $j = 1$ we define $G(1) = 0^{k_{12} \times k_{12}}$.

Based on these quantities one can compute for each coordinate $i = 1, \ldots, k_{11} + k_{12} + r$ of the model average coefficient vector $\hat{\beta}^w$ a valid confidence interval for testing the hypothesis that $H_0: \ \beta_i^w = \beta_{i,0}$. Specifically, it holds (compare Theorem 4.1 of **?**) that

$$T_{n,i} = \sqrt{\frac{N}{\tau_{0i}^2 + \omega_i'(\mathcal{I}^{-1})_{(2,2)}\omega_i}} \left[\hat{\beta}_i^w - \beta_{i,0} - \omega_i' \left(\hat{\beta}_{12}^F - \sum_{j=1}^{2^{k_{12}}} w(j)G(j)\hat{\beta}_{12}^F \right) \right] \tag{5}$$

is asymptotically standard normally distributed under the null hypothesis for $i = 1, \ldots, k_{11} + k_{12} + r$, where $\hat{\beta}_{12}^F$ is the block of the estimated coefficients corresponding to X_{12} in the full model. Based on (5) one can calculate confidence intervals.