This directory contains copyrighted Matlab code for solving the rental machine replacement problem as described in the article by John McClelland, ARA and John Rust, Georgetown University (2018) "Strategic Timing of Investment over the Business Cycle: Machine Replacement in the US Rental Industry" in the Journal of Economics and Statistics Article-DOI: 10.1515/jbnst-2018-0023

The copyright allows for *academic use* of the software but *commercial use is expressly prohibited without the written permission of the authors*. The code provided here solves a dynamic programming problem to determine the optimal replacement timing of machines owned by companies in the machine rental industry. The data on which this analysis was based are confidential, proprietary and are governed by a non-disclosure agreement that prevents the authors from sharing the underling data, though econometric coefficient estimates from econometric analysis of the data (the econometri models and coefficients are described in the article cited above) can be shared publicly for purposes of scientific replication of the some of results in this article, though unfortunately the non-disclosure agreement prevents us from sharing the raw underlying data so it is not possible to replicate the coefficient estimates using the original data we were provided under the NDA. The dynamic programming model and results based on these coefficients can be replicated using the Matlab code we have provided.

To run the model, you need to have a current version of the matlab software. Start up Matlab in the directory where you have downloaded these files and run the Matlab program "dpsolve.m" by typing "dpsolve" at the command prompt for Matlab. The program will run and produce the following results, and will produce plots of results in 7 figures. PDF versions of these 7 figures are included in the figures directory.

If you have any questions or would like to request permission for commercial application of this software, please contact John Rust at Georgetown University Department of Economics, jr1393@georgetown.edu or John McClelland at the American Rental Association (ARA), John.McClelland@ararental.org

```
>dpsolve
Dynamic programming solution for optimal replacement policy for Excavator
(M4) for company A in Southwest

Taxes are not modeled in the DP solution
Replacement strategy is restricted to replacing an old machine with a
brand new one


 Step  1, change in v = 5922342.505270601  (change from 1 contraction
step 37038.973761075)
 Step  2, change in v = 5764563.255806452  (change from 1 contraction
step 4390007.648561018)
 Step  3, change in v = 622065.600913750  (change from 1 contraction step
42281.347733212)
 Step  4, change in v = 95987.646659679  (change from 1 contraction step
26968.341843666)
 Step  5, change in v = 19303.771725972  (change from 1 contraction step
12792.627387522)
 Step  6, change in v = 4970.874342529  (change from 1 contraction step
4396.981950110)
 Step  7, change in v = 5150.586483281  (change from 1 contraction step
2726.435736727)
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 Step  8, change in v = 1261.752276612  (change from 1 contraction step
1004.901057116)
 Step  9, change in v = 655.822395936  (change from 1 contraction step
655.822395936)
 Step 10, change in v = 282.874774137  (change from 1 contraction step
282.874774137)
 Step 11, change in v =  0.000000000  (change from 1 contraction step
0.000000000)

Policy iteration converged in 11 steps, change in v =  0.0000
percentage gain: optimal policy relative to status quo, 1 month old
machine
January 0.032818  0.0327939  0.0327854
February 0.0326246  0.032598  0.0325987
March 0.0324049  0.0323858  0.0323884
April 0.0325731  0.0325524  0.0325603
May 0.0327064  0.0326807  0.032684
June 0.0328397  0.0328142  0.0328128
July 0.0329613  0.0329357  0.0329356
August 0.0330309  0.033004  0.0330018
September 0.0330795  0.0330518  0.0330495
October 0.0330838  0.0330562  0.0330522
November 0.0330477  0.0330255  0.0330228
December 0.0329498  0.0329209  0.0329219

Summary:  optimal policy (month 1, average all macro states and months)
Optimal policy   :  average value: 607390    monthly: 3532.79
percentage gain: 3.28264
Status quo policy:  average value: 588085    monthly: 3420.51

Value to OEC ratio:  optimal: 4.27841  status quo 4.14243
percentage gain: optimal policy relative to status quo, 80 month old
machine
January 0.0670088  0.0670368  0.0373089
February 0.0784072  0.0777759  0.040154
March 0.0716741  0.0650365  0.0371359
April 0.0724766  0.0680392  0.0394878
May 0.0692404  0.0624077  0.0371162
June 0.0740332  0.0680158  0.0398562
July 0.0747217  0.0686958  0.0394563
August 0.0783834  0.0742534  0.0473653
September 0.0678028  0.061123  0.0372291
October 0.0728874  0.0674564  0.0379285
November 0.0798007  0.0740732  0.0450728
December 0.0718181  0.07068  0.0432721

Summary:  optimal policy (month 80, average all macro states and months)
Optimal policy   :  average value: 537246    monthly: 3124.81
percentage gain: 6.0446
Status quo policy:  average value: 506623    monthly: 2946.7

Value to OEC ratio:  optimal: 3.78432  status quo 3.56862
```